MAGELLAN PROJECT

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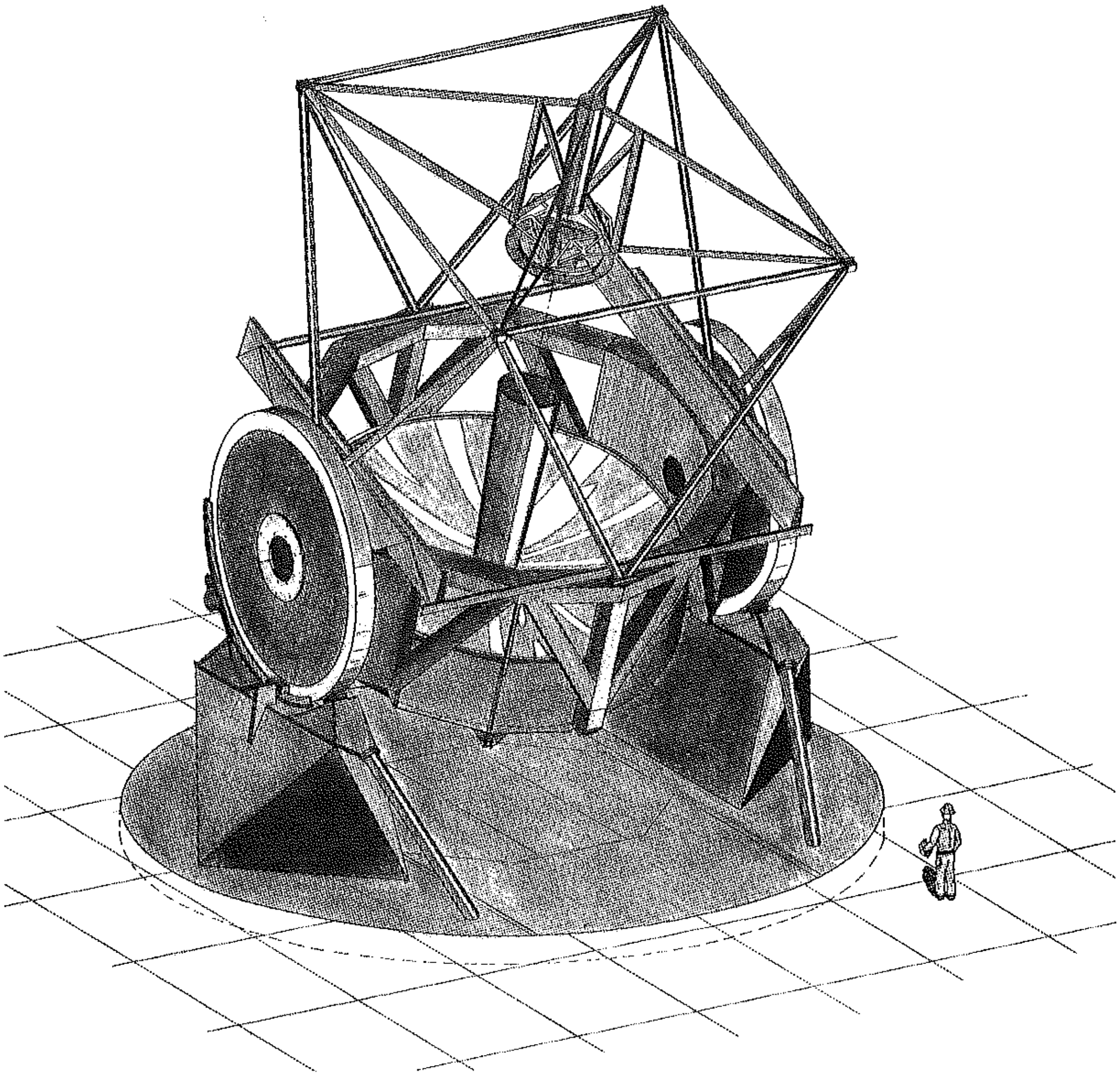

Dynamics for Two-Motor, One-Encoder Drives

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INTRODUCTION AND SUMMARY

This report analyzes some of the dynamics involved in using two motors to drive the axes of an alt-az mounted telescope. A significant flexibility is considered to exist between the two motors as might be the case for the altitude axis. A simple control scheme is investigated wherein the torque applied by one motor is a constant multiple of the other, and the position is encoded near one of the motors.

The results show that there is a critical value (a singularity) for the multiplying constant. At the critical value, the flexibility of the structure becomes (mathematically) unobservable and does not enter into the dynamics. At less than the critical value, the system displays the relatively benign condition in which an antiresonance is followed at a higher frequency by a resonance*. For a multiplier greater than the critical value, the system becomes much more difficult to handle, with the resonance preceding the antiresonance. This condition necessarily involves a large phase lag near the frequency of resonance, which is the frequency of the "free-boundary" mode of vibration.

In practice, the multiplying constant would be determined through measurements made on the finished structure and would be made slightly less than the critical value. This would result in acceptable phase behavior.

DESCRIPTION OF THE MODEL

A model is used consisting of two inertias connected by a spring as shown in figure 1. Two input torques are applied to the system, and the output position is taken at one inertia. The model is a simplified representation of a wide range of mechanical systems and allows the important frequency domain features to be examined. This model neglects other resonances that would be present in the structure.

The size of the inertias and the value of the spring constant are not constrained to any particular values, although for the sake of clear illustration all these values have been set to unity. The variable parameter in the model is alpha, the ratio of the two torques applied to the structure. As this ratio is varied, the qualitative behavior of the system changes as will be described in the next section.

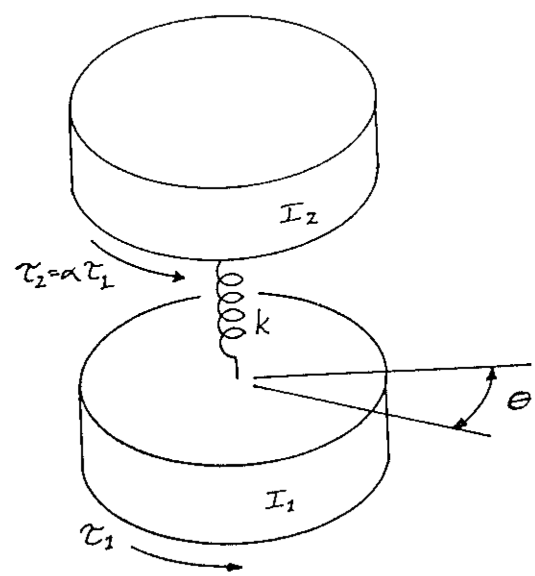


Figure 1:
One spring and two rotational inertias comprise the model system. Torque is applied to both inertias with the torque applied to I_2 being a constant multiple of the torque applied to I_1. The output variable is the position of I_1.

* A resonance appears as a peak in the frequency response, whereas an antiresonance appears as a valley.

DESCRIPTION OF THE RESULTS

Figures 2 and 3 show the magnitude and phase responses of the system as the torque ratio, alpha, is varied. The mathematics behind the generation of these curves is detailed in the Appendix A.

The important feature is the change in the phase characteristics as the antiresonance goes from preceding to following the resonance. The phase plot in figure 3 shows an area of phase lead when the antiresonance precedes the resonance, and an area of phase lag when it follows. This occurs as the value for alpha goes from smaller to larger values. For the illustrated system, the critical value for alpha is unity. At this value, the resonance and antiresonance coincide (and disappear).

The magnitude plot in figure 2 shows another interesting feature. As alpha is varied and the frequency of the antiresonance changes, the frequency of the resonance remains fixed. This fixed resonant frequency is the natural frequency in which the two masses are unconstrained (free boundaries) and simply "bounce against each other". This fact is apparent directly from the magnitude plot given the assumed values of unity for the inertias and spring rate. It is also apparent in a more general fashion in the mathematics in, Appendix A.

IMPLICATIONS OF THE RESULTS

It is hard to overstate the importance of the phase in determining the system performance. If the phase makes steep transitions to large negative values, the problem becomes particularly vexing. In such a case, a practical system can be made stable only if the bandwidth is well below the frequency of the sharp phase transition or if the servo controller is considerably more complex (i.e. expensive and touchy). It is for this reason that the case in which an antiresonance follows a resonance is a significant concern.

A low frequency antiresonance well before the resonance is also undesirable, but the consequences are not nearly as adverse. As a practical matter, it would result in disturbance rejection that is a bit poorer than it would otherwise be. The antiresonance itself will not however limit the servo bandwidth nor will it drive a closed-loop servo unstable.

Given that the free-boundary resonance frequency is fixed, there may be other lower natural frequencies in the real structure that limit the servo performance. If this is the case, this resonant mode is less important regardless of whether it precedes or follow the antiresonance.

CONCLUSIONS

If the free-boundary mode of vibration is well above the desired servo bandwidth--say, by at least a factor of ten--then it is of very little concern.

If this mode is close to the desired servo bandwidth, then the ratio of torques applied by the two motors must be set to insure acceptable behavior of the gain and phase characteristics.

SUGGESTIONS FOR FURTHER CONSIDERATION

Additional position feedback combined with a more complicated servo controller would permit better control of the flexible mode addressed in this report. This means of servo control should be the topic of a separate report.

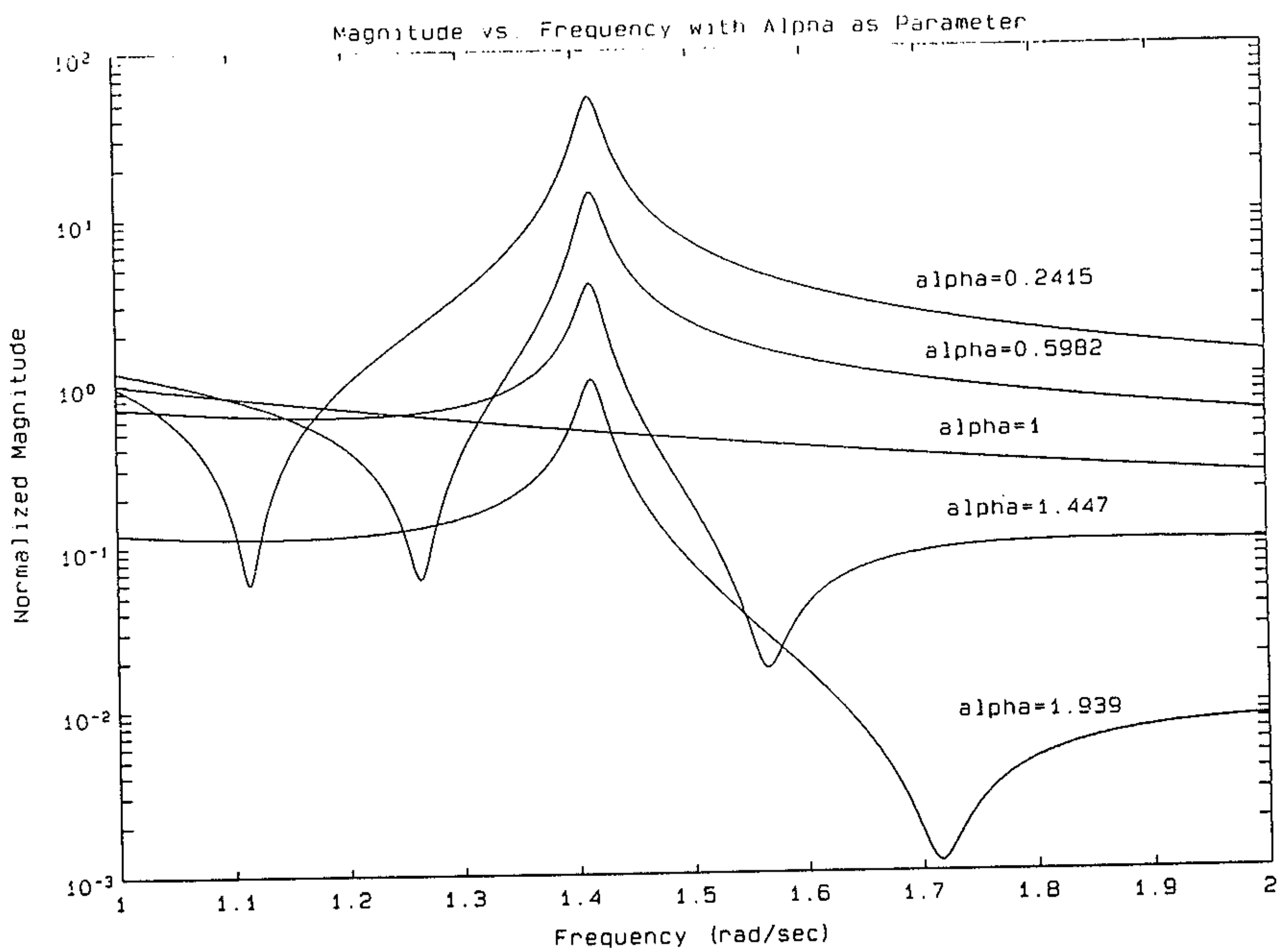


Figure 2:
As the torque ratio alpha, is varied from low to high values, the antiresonance passes through the resonance as it moves from left to right. At alpha = 1, the peak and valley coincide and cancel. This represents a system in which the flexible modes could not be excited. Note also that the frequency of the resonant peak stays fixed. Only the frequency of the antiresonance changes. These plots have been offset in the vertical direction in order to make them more distinct. A small amount of damping has also been added to keep the peaks and valley finite.

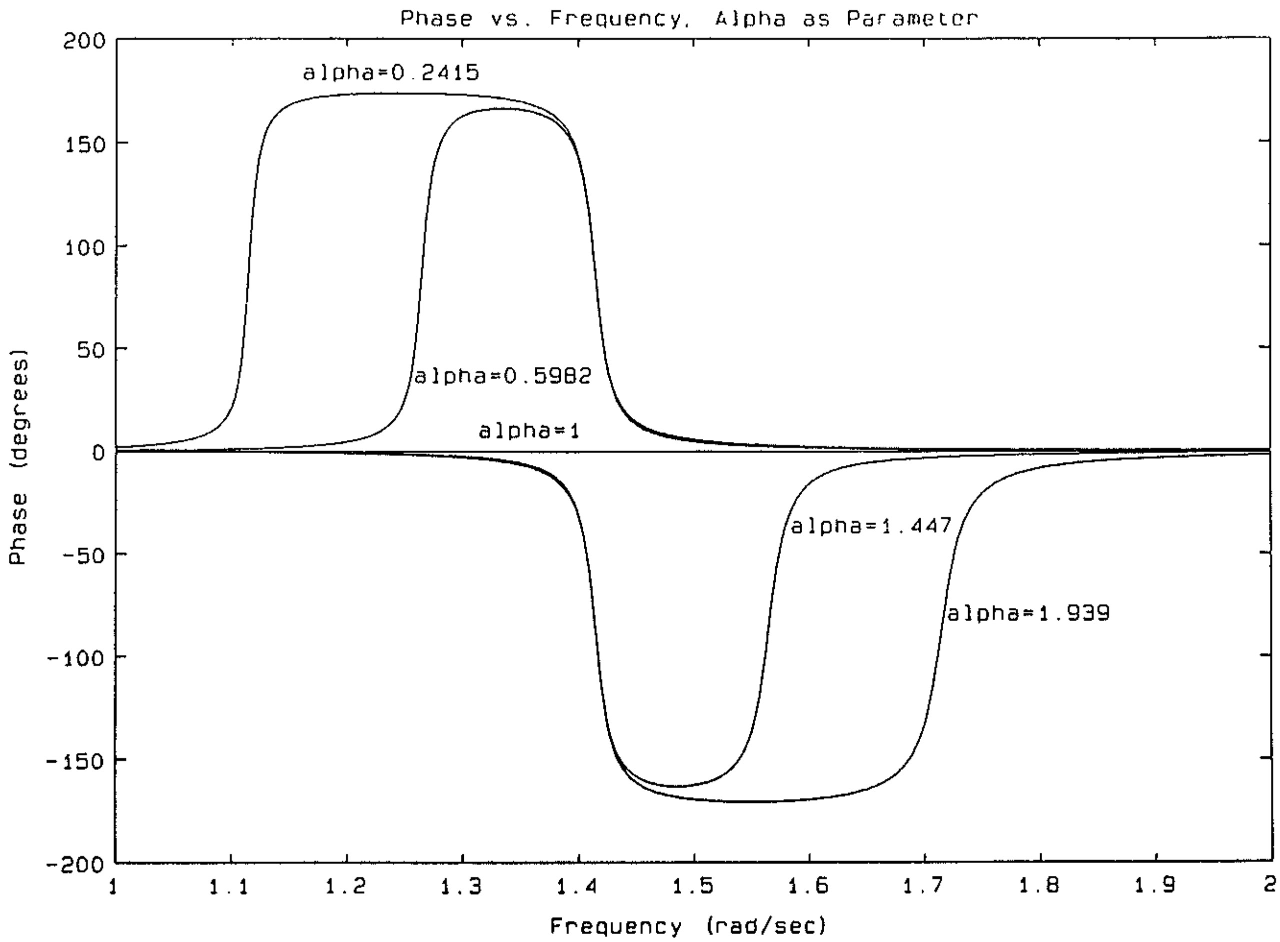
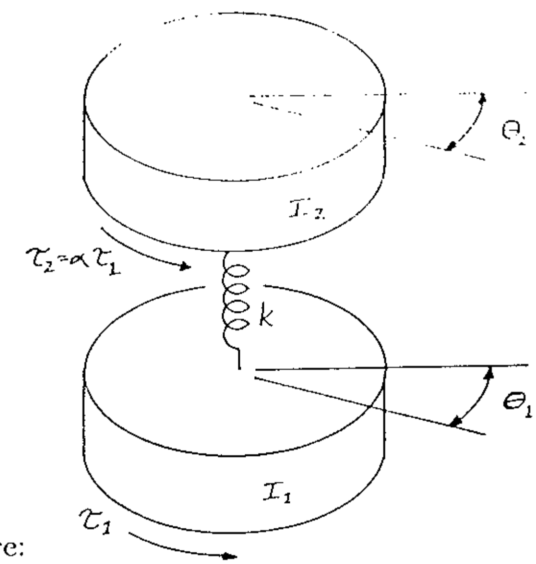


Figure 3:
As the torque ratio alpha is varied from low to high values, the system displays a region of phase lead that gives way to a region of phase lag. At alpha = 1, the area of phase lead or lag is zero. Important here are the sharp transitions to large negative phase angles at the frequency of the resonant peaks (shown in Figure 2).

APPENDIX A

Supporting Analysis



Let $\tau_2 = \alpha\tau_1$. The equations of motion are:

$$\tau_1 + \kappa(\theta_2 - \theta_1) - I_1\ddot{\theta}_1 = 0$$

$$\alpha\tau_1 + \kappa(\theta_1 - \theta_2) - I_2\ddot{\theta}_2 = 0$$

Taking the Laplace transform of (1) and (2) and combining yields

$$\frac{\Theta_1}{\Upsilon_1} = \frac{I_2 s^2 + \kappa(\alpha + 1)}{s^2(I_1 I_2 s^2 + \kappa(I_1 + I_2))}$$

where Θ_1 and Υ_1 are the transformed time functions θ_1 and τ_1, and s is the transform variable. Θ_1/Υ_1 has zeros (zeros of the numerator) at

$$s_Z = \pm j\sqrt{\frac{\kappa(\alpha + 1)}{I_2}}$$

and has poles (zeros of the denominator) at

$$s_p = 0, 0, \pm j\sqrt{\frac{\kappa(I_1 + I_2)}{I_1 I_2}}$$

Note that Sp is not a function of α. Neglecting the two poles at 0 (which correspond to rigid body motion) and considering only the poles and zeros that result from flexible modes, if

$$(\alpha + 1) = \frac{(I_1 + I_2)}{I_1}$$

then

$$|s_Z| = |s_P|$$

and the resonant modes cancel exactly. If

$$(\alpha + 1) < \frac{I_1 + I_2}{I_1}$$

then

$$|S_z| < |S_p|$$

and the system will show a region of phase lead. Finally, if

$$(\alpha + 1) > \frac{I_1 + I_2}{I_1}$$

then

$$|S_z| > |S_p|$$

and the system will show a region of phase lag.

APPENDIX B

Plot Generating Program

```
% This file runs in matlab.
% Calculates and plot frequency responses for a two-inertia, one-spring system
% that is driven at both masses with a torque and encoded at only one position.
% Alpha is the variable ratio of the noncolocated torque to the colocated
% torque.  Inertias and spring rate are unity.

 if exist('mag')~=1
        alpha=[ 0.2415 0.5982 1 1.4468 1.9385]';
        zeros=[i*sqrt(alpha+1) -i*sqrt(alpha+1)]';
        zeros=zeros-(0.005)*(zeros .* conj(zeros));
        poles=[0 0 i*sqrt(2) -i*sqrt(2)]';
        poles=poles-(0.005)*(poles .* conj(poles));
        gain=ones(zeros(1,:));
        w=linspace(1,2,400);
        [num,den]=zp2tf(zeros,poles,gain);
        [mag,phase]=bode(num,den,w);
 end
 semilogy(w,mag(:,1)*4,w,mag(:,2)*2,w,mag(:,3),w,mag(:,4)/2,w,mag(:,5)/16)
 title('Magnitude vs. Frequency with Alpha as Parameter')
 [f1,m1]=ginput(1);
 text(f1,m1,['alpha=',num2str(alpha(1))]), pause
 [f2,m2]=ginput(1);
 text(f2,m2,['alpha=',num2str(alpha(2))]), pause
 [f3,m3]=ginput(1);
 text(f3,m3,['alpha=',num2str(alpha(3))]), pause
 [f4,m4]=ginput(1);
 text(f4,m4,['alpha=',num2str(alpha(4))]), pause
 [f5,m5]=ginput(1);
 text(f5,m5,['alpha=',num2str(alpha(5))])
 xlabel('Frequency (rad/sec)')
 ylabel('Normalized Magnitude')
 pause

 A=input('Save the plot? Y/N [N]:','s');
 if isempty(A)
         A='N';
 end

 if A~='N'
         meta d:\matlab\two-drv\twodrvfr
 end

 plot(w,phase(:,1:2)+180,w,phase(:,3:5)-180)
 [f1,p1]=ginput(1);
 text(f1,p1,['alpha=',num2str(alpha(1))]), pause
 [f2,p2]=ginput(1);
 text(f2,p2,['alpha=',num2str(alpha(2))]), pause
 [f3,p3]=ginput(1);
 text(f3,p3,['alpha=',num2str(alpha(3))]), pause
 [f4,p4]=ginput(1);
 text(f4,p4,['alpha=',num2str(alpha(4))]), pause
 [f5,p5]=ginput(1);
 text(f5,p5,['alpha=',num2str(alpha(5))])
 title('Phase vs. Frequency, Alpha as Parameter')
 xlabel('Frequency (rad/sec)')
 ylabel('Phase (degrees)')
 pause

 A=input('Save the plot? Y/N [N]:','s');
 if isempty(A)

         A='N';
 end

 if A~='N'
         meta d:\matlab\two-drv\twodrvfr
 end
```